The Precession of a Spinning Spacecraft Due to Radiation Pressure Torque

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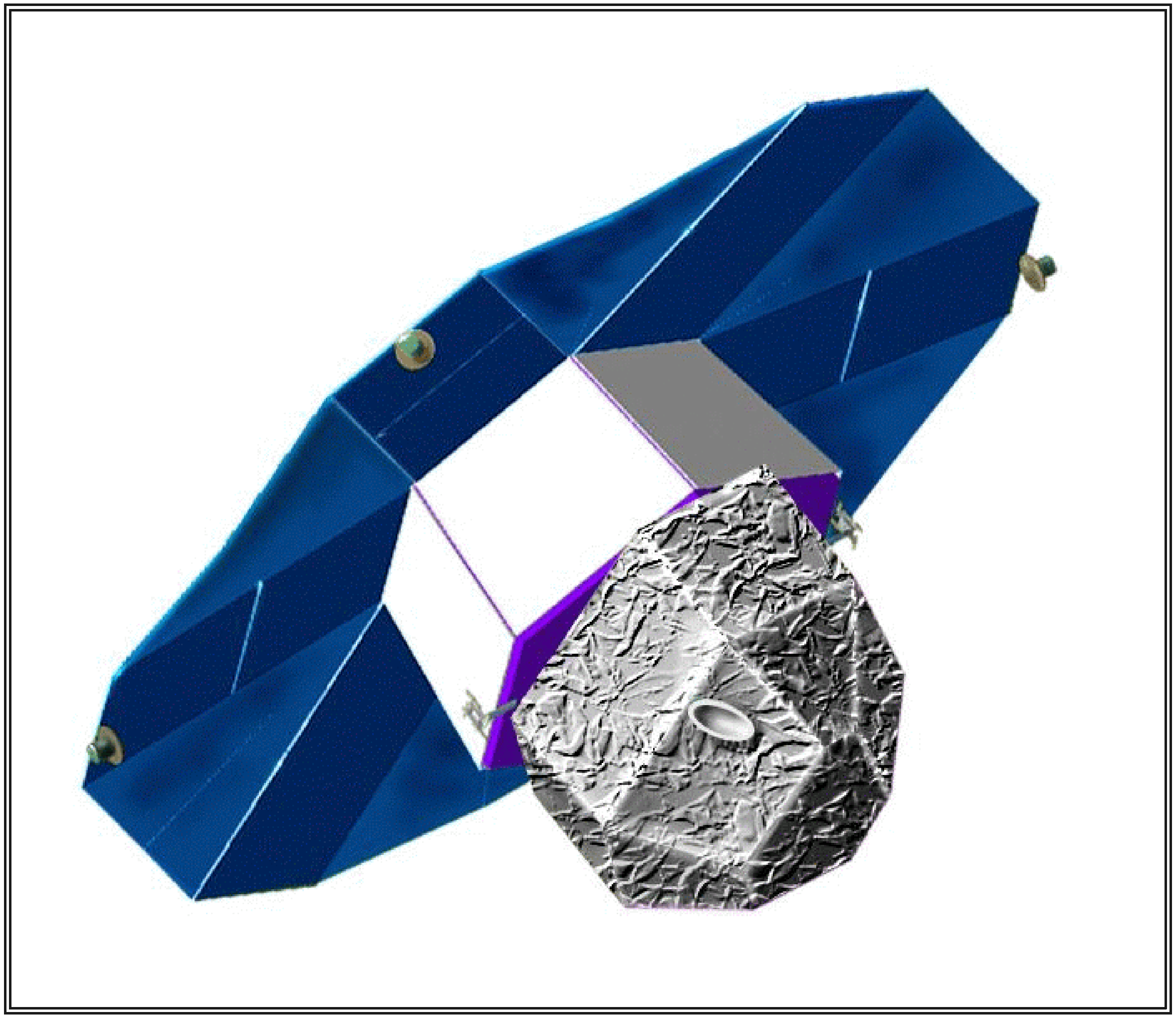
The FAME Mission

Full-Sky Astrometric Mapping Experiment

- Selected for Phase-A study in MIDEX competition.
- Collaboration among: USNO, SAO, Lockheed, NRL, Omitron, and IPAC.
- Principal Investigator: Kenneth J. Johnston, USNO.
- Hipparcos tradition.
    - Two apertures perpendicular to the spin direction.
    - Scanning pattern: spin axis precessed around the Sun direction.

Present Spacecraft



Importance of Smooth Rotation

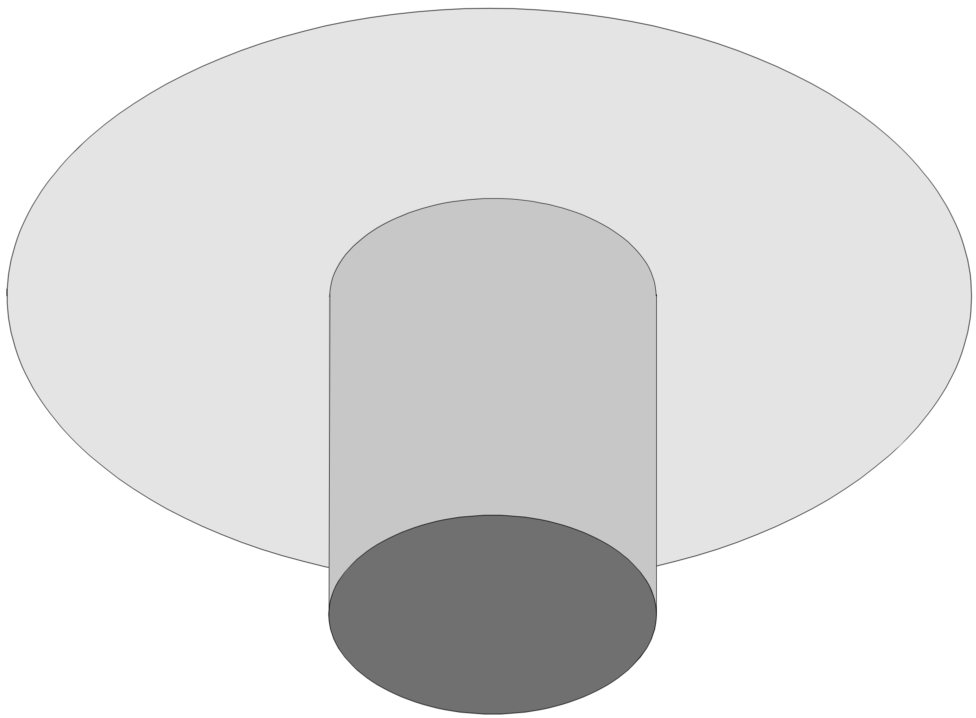
Case*	$\sigma(\Delta\phi) / \sigma_0$
Short Spans (6 ACE Per Rotation)	3.03
Long Spans (1 ACE Per Rotation)	0.37
No ACE, 16 ϕ Coefficients Per Rotation	0.13
No ACE, 4 ϕ Coefficients Per Rotation	0.106
No ACE, 4 ϕ Coefficients Per Rotation, No Parameters for ν or ξ.	0.037

Assumes: Field of view = 1.6 deg.
Rotation period = 30 min.
Precession = 0.5 deg per rotation.
Batch interval = 6 rotations.
Observations per rotation = 6900.
Stars observed = 3000.
(The larger number of stars available could make $\sigma(\Delta\phi)$ smaller.)

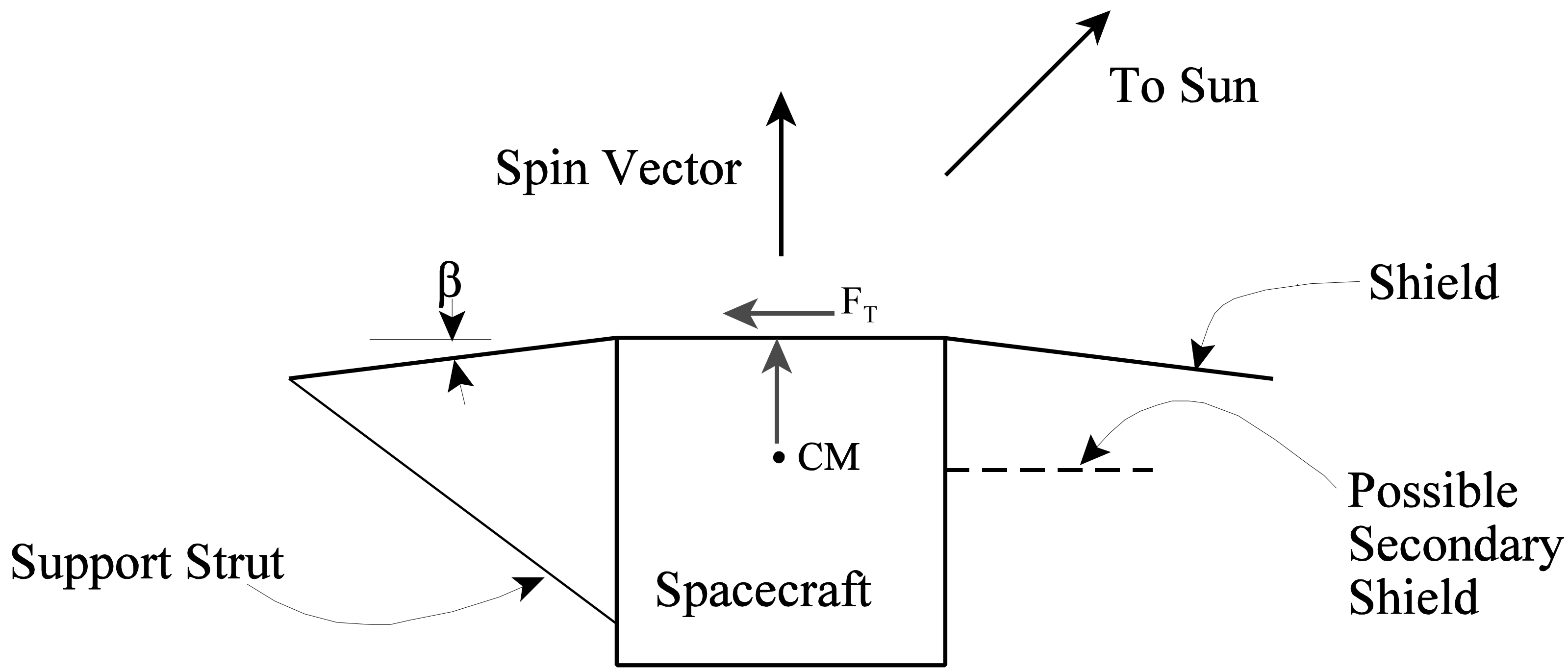
* From Chandler and Reasenberg, A New Approach to Post-Hipparcos Astrometric Surveys from Space, in preperatiom, 1999.

Precession Driven by Radiation Pressure

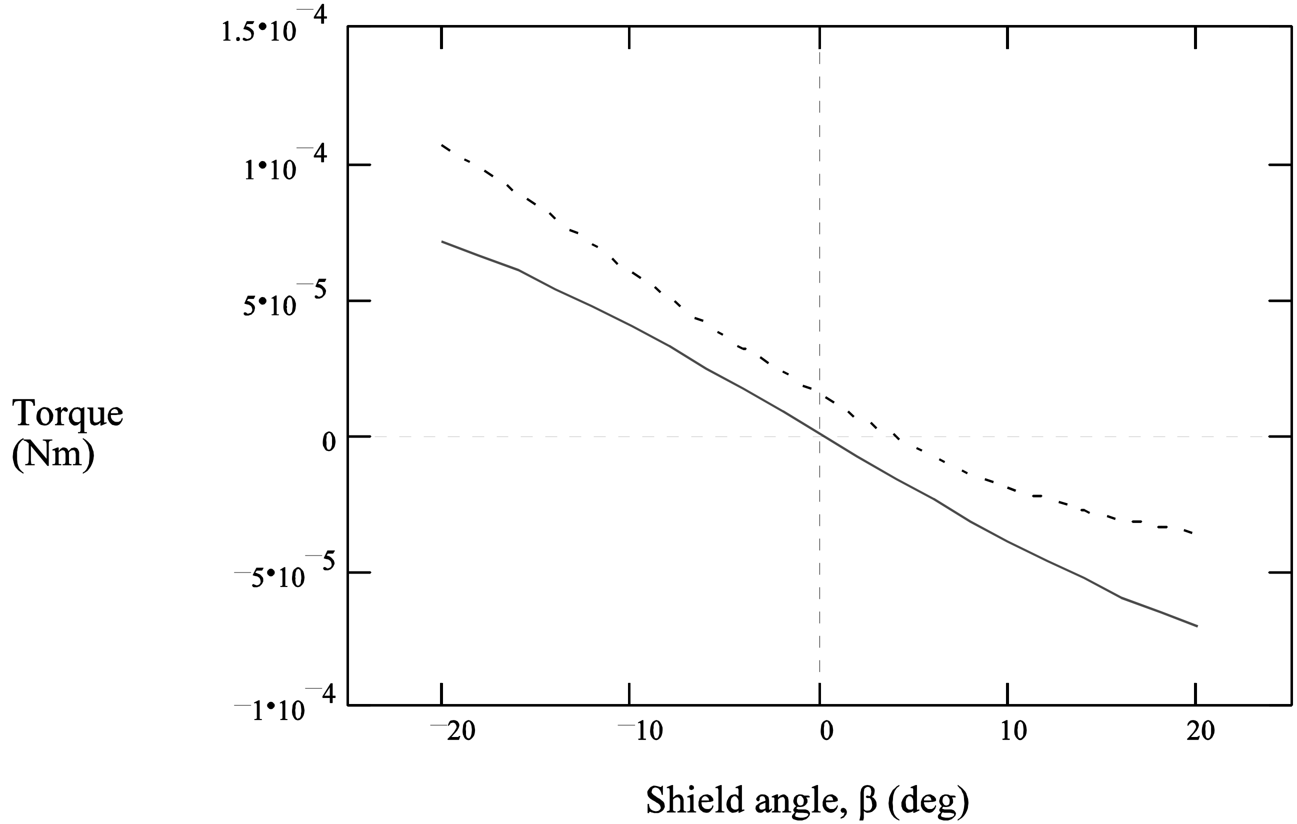
- Initial realization: November 1996
- Presented: TM97-03 (6/97), SPIE-3356 (3/98), DDA98 (4/98, Murison)



Geometry of Radiation Torque



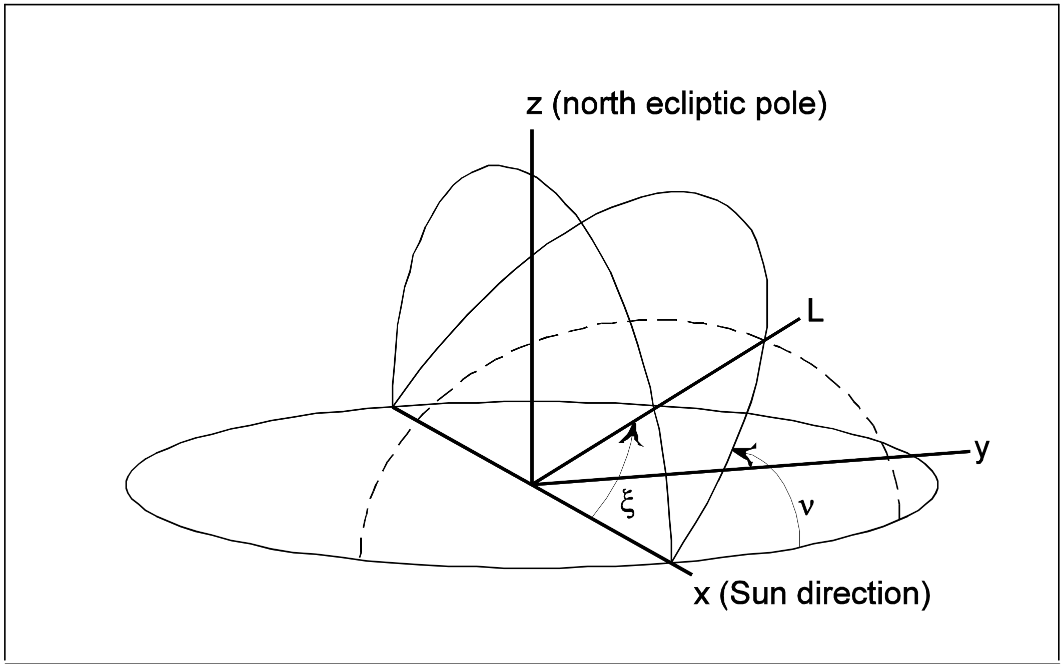
Variation of Torque with Sweep Angle



The Celestial Mechanics Problem

- Trivial for fixed Sun direction.
- For the analysis of Mission data, use numerical integration of angular ephemeris.
  - Must include nutation and complex (torque) drive.
  - Integrate variational equations (for use of WLS estimator).
- Find low-order analytic solution.
  - Yields understanding of problem.
  - Provides check on numerical integration.

Equations of Motion in Rotating Coordinates



Spherical geometry of precession with ξ = 45 deg and ν = 60 deg. The dashed line shows the trajectory of the spacecraft angular momentum vector on the celestial sphere. The view is from 15 deg above the reference plane and 20 deg to the left of the x axis.

Equations of Motion in Rotating Coordinates, II

Angular momentum, L, in frame rotating by ω

$$N_R = N_I = \left(\frac{dL}{dt}\right)_I = \left(\frac{dL}{dt}\right)_R + \omega \times L$$

Approximate torque, N

$$N = N_0 \sin\xi \cos\xi \,(0, -\sin\nu, \cos\nu)$$

where N_0 depends on the area and optical properties of the shield and the sweep back angle, β. Note that torque actually depends on ξ^* between surface and Sun direction.

Equations of Motion in Rotating Coordinates, III

$$\dot{\xi} = -\omega \cos\nu$$

$$\dot{\nu} = \frac{N_0}{L} \cos\xi \left(1 + \alpha \frac{\sin\nu}{\sin\xi} \right)$$

where α is a small parameter

$$\alpha = \frac{\omega L}{N_0}$$

[α is the precession period in years × $\cos(\xi_0)$ $\alpha \approx 0.02$ for 10 day precession]

Analytic Solution, I

For $\alpha = 0$ (i.e., $\omega = 0$), the solution is trivial

$$\xi = \xi_0$$

$$\nu = \nu_0 + \Omega\, t$$

$$\Omega = \frac{N_0 \cos(\xi_0)}{L}$$

(But how do we make the Sun stop moving around the Earth?)

Analytic Solution, II

If you have not a solution, assume it

$$\xi = \xi_0 - \xi_1 \sin(\Omega t + \varepsilon_\xi)$$

$$\nu = \nu_0 + \Omega t - \nu_1 \cos(\Omega t + \varepsilon_\nu)$$

Heuristic reason for the form of the trial solution:

Looking toward the Sun from the spacecraft, with the north ecliptic pole called up, the Sun is seen (in an inertial frame) to move to the left as a result of Earth's annual motion. Assume the spin vector will precess clockwise around the Sun in the rotating coordinate system that has the Sun along the x axis. As the spin vector is rising through the ecliptic (to the left of the Sun), the Sun is moving to decrease ξ: $\dot{\xi}$ is at a minimum. Next consider the spin vector at its maximum height above the ecliptic. It is moving to the right as the Sun moves to the left: $\dot{\nu}$ is at a maximum. These results are consistent with the trial solution if ξ_1 and ν_1 are positive and ϵ_ξ and ϵ_ν are approximately equal to ν_0, as will be shown below to be one solution.

Analytic Solution, III

$$\cos\xi = \cos\xi_0 [J_0(\xi_1) + 2J_2(\xi_1)\cos(2\Omega t + 2\varepsilon_\xi)] + 2\sin\xi_0 J_1(\xi_1)\sin(\Omega t + \varepsilon_\xi) + \ldots$$

$$\sin\xi = \sin\xi_0 [J_0(\xi_1) + 2J_2(\xi_1)\cos(2\Omega t + 2\varepsilon_\xi)] - 2\cos\xi_0 J_1(\xi_1)\sin(\Omega t + \varepsilon_\xi) + \ldots$$

$$\begin{aligned}\cos\nu &= \cos(\nu_0 + \Omega t)[J_0(\nu_1) - 2J_2(\nu_1)\cos(2\Omega t + 2\varepsilon_\nu)] \\ &\quad + 2\sin(\nu_0 + \Omega t)J_1(\nu_1)\cos(\Omega t + \varepsilon_\nu) + \ldots\end{aligned}$$

$$\begin{aligned}\sin\nu &= \sin(\nu_0 + \Omega t)[J_0(\nu_1) - 2J_2(\nu_1)\cos(2\Omega t + 2\varepsilon_\nu)] \\ &\quad - 2\cos(\nu_0 + \Omega t)J_1(\nu_1)\cos(\Omega t + \varepsilon_\nu) + \ldots\end{aligned}$$

$$J_0(w) = 1 - \frac{w^2}{4} + \frac{w^4}{64}$$

$$J_1(w) = \frac{w}{2} - \frac{w^3}{16}$$

$$J_2(w) = \frac{w^2}{8} - \frac{w^4}{96}$$

Analytic Solution, IV

[plug and play]

$$\Omega = \frac{N_0}{L} \cos \xi_0$$

$$\xi = \xi_0 - \frac{\alpha}{\cos(\xi_0)} \sin(\Omega t + \nu_0)$$

$$\nu = \nu_0 + \Omega t - \frac{\alpha}{\sin(\xi_0) \cos^2(\xi_0)} \cos(\Omega t + \nu_0)$$

This is the lowest order solution.

For ξ_0 = 45 deg, $\nu_1 = 2\xi_1$.

Next Order Analytic Solution

[this takes longer]

$$\Omega = \frac{N_0}{L} \cos \xi_0 (1 - \kappa \alpha^2)$$

$$\kappa = \frac{1}{4 \cos^2(\xi_0)}$$

For ξ_0 = 45 deg, κ = 0.5

$$\alpha = \frac{\omega L}{N_0} \approx 0.02$$

Numerical Solution, I

$$\dot{\xi} = -\omega \cos\nu$$

$$\dot{\nu} = \frac{N_0}{L} \cos\xi \left(1 + \alpha \frac{\sin\nu}{\sin\xi} \right)$$

Solve for $\xi_0 = 45$ deg and $\nu_0 = 0$

$$\nu(t) = \nu_0 + \Omega t - \frac{\alpha}{\sin(\xi_0)\cos^2(\xi_0)} \cos(\Omega t + \nu_0)$$

$$\nu(0) = -\nu_1 \approx -\frac{\alpha}{\sin(\xi_0)\cos^2(\xi_0)}$$

Numerical Solution, II

- ν_1 was found to lowest order in α.

- Iterative approach needed to find $\nu_1(\alpha)$. (This converges quickly.)

  - Integrate equations of motion.
  - Get Ω from $\xi(t)$ by finding the period of its time variation.
  - Get $\nu_1(\alpha)$ from $\nu(t) - \Omega t$ by finding the amplitude of its time variation.

- Analysis done for $\alpha = \{0.01, 0.02 \ldots 0.06\}$.

- Results for $\xi_0 = 45$ deg.

  - $\kappa(\alpha) = -\frac{1}{2}(1.00010 + 0.01406\,\alpha + 0.65554\,\alpha^2)$
  - $\xi_1(\alpha) = \sqrt{2}\,\alpha\,(0.99998 - 2.3398\,\alpha^2)$
  - $\nu_1(\alpha) = 2\sqrt{2}\,\alpha\,(1.00002 - 1.54885\,\alpha^2)$

Directions and Discussion

- Higher order solution
  - Extend series in α.
  - Include Earth eccentricity.
  - Investigate slow precession -- N per year

- Present solution nearly good enough for FAME data analysis, but physical model must include several other effects and variational equations will be needed.
  - Radiation from Earth.
  - Gravity gradient acting on non-spherical spacecraft.
  - Fluctuations of solar flux (under investigation by Marc Murison).
  - Real charqcteristics of solar shield.

- If FAME does not use solar torque, but thrusters to precess spacecraft.
  - Hipparcos approach has been shown to yield poor model of rotation.
  - Use a pair of thruster firings every 3/2 rotations.
  - Design solar shield for null torque.
  - Spin rate, etc would be determined each time.